MAD-TH-08-05, RUNHETC-2008-03

arXiv:0803.3068v3 [hep-th] 10 Jun 2008

Dynamics of Warped Flux Compactifications

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Abstract

We discuss the four dimensional effective action for type IIB flux compactifications, and obtain the quadratic terms taking warp effects into account. The analysis includes both the 4-d zero modes and their KK excitations, which become light at large warping. We identify an 'axial' type gauge for the supergravity fluctuations, which makes the four dimensional degrees of freedom manifest. The other key ingredient is the existence of constraints coming from the ten dimensional equations of motion. Applying these conditions leads to considerable simplifications, enabling us to obtain the low energy lagrangian explicitly. In particular, the warped Kähler potential for metric moduli is computed and it is shown that there are no mixings with the KK fluctuations and the result differs from previous proposals. The four dimensional potential contains a generalization of the Gukov-Vafa-Witten term, plus usual mass terms for KK modes.

Contents

1 Introduction

Understanding warp effects in string theory is important both for theoretical issues (e.g. gauge/string dualities) and for making more precise four dimensional predictions. The original motivation for warped geometries comes from considering stacks of large numbers of branes, and reveals deep dualities

between supergravities and gauge theories, as in the AdS/CFT correspondence [1]. A well studied duality with $\mathcal{N} = 1$ supersymmetry involves the gauge theory of D5-branes wrapping an isolated two-cycle of a Calabi-Yau compactification (the resolved conifold, see for example [2]). It is dual to a ten dimensional supergravity solution found by Klebanov and Strassler [3]. In this example it is clear that nontrivial variation of the warp factor is related to the nonzero beta function of the theory, while the deformation of the conifold is associated to the mass gap. Some analysis of fluctuations around this background has been made [4, 5, 6, 7].

Warped geometries also play an important role in supersymmetry breaking scenarios from string theory. For instance, placing antibranes at the end of the conifold, [8] found a supergravity dual of a nonsupersymmetric field theory. Other approaches include brane-antibrane systems [9, 10], and various other geometrical effects [11, 12]. There have also been recent developments in metastable vacua, following the work of [13]; see, for example [14]-[17]. Overall, these works suggest that strongly warped supergravities which break supersymmetry are dual to dynamical supersymmetry breaking in gauge theories.

These works studied the local geometry near small cycles in the compact manifold, as this is what is relevant for gauge-gravity duality. The global study of warped compactification is far more difficult, as one cannot find exact solutions in this case. Nevertheless one can make progress by combining a well chosen ten dimensional ansatz with results obtained from some corresponding four dimensional effective field theory. In particular, one can look for solutions obtained from a six dimensional Calabi-Yau manifold, by turning on additional fields, and adding conformal and warp factors to the metric. While explicit Calabi-Yau metrics are not known, a great deal of technology has been developed to compute low energy observables anyways, which might be adapted to these warped solutions.

A prototype here is the work of Giddings, Kachru, and Polchinski (GKP) [18], who considered type IIB flux backgrounds satisfying certain BPS-type conditions (see [19, 20, 21, 22] for reviews and [23, 24, 25] for earlier work). They were able to derive many properties of the dimensionally reduced theory, including the flux superpotential and the generation of large hierarchies. An important lesson from GKP is that the dimensionally reduced action encodes many features of the full theory in a simple way.

The actual results of GKP did not include the warp corrections to the four dimensional effective action. While this is consistent in the limit of large volumes or small fluxes, clearly one would like to go on and derive these corrections.

To begin with some general comments, the warp factor is not holomor-

phic and thus one expects it not to affect holomorphic quantities such as the superpotential and gauge kinetic terms.[1] However, in general it will affect non-holomorphic quantities such as the Kähler potential. Furthermore, since the warp factor is produced by backreaction from fluxes and branes, one cannot hope to find a general formula for the Kähler potential purely within the moduli sector; rather it must couple moduli and other matter. One approach is to keep the warp factor as a six-dimensional field which is determined by the fluxes and matter by solving a six-dimensional equation, and then use it in defining the 4d effective Kähler potential. If this 6d equation uniquely determines the warp factor, we can still regard the formalism as a 4d effective action.

A first attempt at a quantitative analysis was made in [29], where a conjecture was made for the effect of warping on the complex structure moduli space metric (see also [30] for an earlier study of warped moduli space). Further study of the fluctuating modes was made in [26, 31, 32, 33]. However, these analyses became extremely involved due to mixings between the various ten dimensional modes and the need for "compensator fields," and simple results were not obtained. For example, the validity of the conjecture of [29] remained unclear. This was further studied in the language of generalized complex geometry in [28].

Another fundamental difficulty is that, given strong warping, KK modes can acquire masses of the same scale as the stabilized complex moduli, so they may need to be included in an effective description. At present, a consistent effective action is unknown, and there are doubts regarding the validity of a four dimensional description [26, 33, 34].

The aim of our work is to perform a consistent dimensional reduction in the presence of warping, including both the 4d zero modes and the first KK excitations. The main obstacles which will be faced are finding a well-defined compactification procedure (we will point out the subtleties in a moment), and identifying the correct 4d degrees of freedom from the highly coupled 10d fluctuations. Before starting, we review the warped supergravity backgrounds which will be considered, and then summarize our work.

[1] A caveat here is that the 4d superspace formulation of warped compactifications is not yet fully understood. It has been suggested [26] that some of the holomorphic Kähler coordinates should be expressed as periods of warped forms. See also [27, 28].

1.1 Review of GKP ansatz

Following [18], one starts with the bosonic action

$$S_{IIB} = S_{EH} + S_{matter} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\left\{R - \frac{\partial_M\tau\partial^M\bar{\tau}}{2(\mathrm{Im}\tau)^2} - \frac{G_3\cdot\bar{G}_3}{12\mathrm{Im}\tau} - \frac{\tilde{F}_5^2}{480}\right\} +$$
$$- \frac{i}{8\kappa_{10}^2}\int\frac{C_4\wedge G_3\wedge\bar{G}_3}{\mathrm{Im}\tau} + S_{loc}. \tag{1.1}$$

We take space-time to be a warped product of $\mathbb{R}^{3,1}$ with a six dimensional Kähler manifold M. Let $\tilde{g}_{mn}$ be a Ricci-flat Kähler metric on M, then the metric ansatz is

$$ds_{10}^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n\,. \tag{1.2}$$

The field strengths are chosen to preserve Lorentz invariance and self-duality is imposed on the five form,

$$G_3 = \frac{1}{6}G_{mnp}(y)dy^m dy^n dy^p = F_3 - \tau H_3\,, \tag{1.3}$$

$$\tilde{F}_5 = \partial_m\alpha(y)(1+\star)dy^m dx^0 dx^1 dx^2 dx^3\,. \tag{1.4}$$

Since we are interested in flux compactifications, we will assume that F_3 and H_3 are three-forms in nontrivial classes of $H^3(M,\mathbb{Z})$.

The warp factor is of the general form

$$e^{-4A(y)} = c + e^{-4A_0(y)} \tag{1.5}$$

where the dimensionless parameter c is related to the total volume by $V_{CY}\sim\alpha'^3\,c^{3/2}$, and A_0 is produced by matter sources (fluxes or branes). The large volume limit $c \gg e^{-4A_0}$ corresponds to a small number of fluxes or branes, where backreaction may be ignored. In the present work we will address the general case, which includes the strongly warped limit $c \ll e^{-4A_0}$. Notice also that in this regime the c fluctuation does not coincide with the usual parameterization of the universal Kähler modulus as an overall scaling of the underlying CY [26].

Furthermore, we will concentrate on the BPS backgrounds of [18], which are analogous to the brane metrics of AdS/CFT. These satisfy

$$\alpha = e^{4A}\;,\;\; \frac{1}{4}(T^m_m - T^\mu_\mu)^{loc} = T_3\rho_3^{loc}\,. \tag{1.6}$$

As a result, the equations of motion are verified automatically if the three form flux is imaginary self-dual (ISD):

$$\star_6\,G_3 = iG_3\,. \tag{1.7}$$

Similar conditions may be written for backgrounds sourced by anti-branes, with the flux becoming IASD. As pointed out by GKP, the ISD condition fixes the complex structure moduli and leads to a constant background dilaton.

Small fluctuations around this background should be described by a 4d $\mathcal{N} = 1$ supergravity effective action. A reasonable starting point for this is to take the $\mathcal{N} = 2$ supergravity obtained by KK reduction of IIb supergravity on M, and then apply an orientifold projection. This is expected to be a good description in the large volume limit, with small quantum corrections. Furthermore, we are assuming that backreaction can be ignored and thus the warp factor is essentially constant. This will be true if we take the limit $\alpha' \to 0$ while holding the number of flux units N fixed. Conversely, large N dualities or large hierarchies arise, even in the $\alpha' \to 0$ limit, when $\alpha' N$ is held fixed.

In this limit, the 4d Kähler potential for the metric moduli takes the well-known form,

$$K = -3\log(-i(\rho+\bar{\rho})) - \log\big(-i\int J\wedge J\wedge J\big) - \log\big(-i\int_M \Omega\wedge\bar{\Omega}\big)\,. \quad (1.8)$$

Fluxes generate a scalar potential for the complex moduli,

$$\mathcal{V} = -\frac{1}{2\kappa_{10}^2 \mathrm{Im}\tau}\int_M G_3\wedge(\star_6\bar{G}_3 + i\bar{G}_3) \quad (1.9)$$

which vanishes in the ISD case. From the scalar potential and Kähler potential we can infer that the superpotential for the complex moduli is of the Gukov-Vafa-Witten type [18, 36, 23, 37],

$$W_{GVW} = \int_M \Omega\wedge G_3 \quad (1.10)$$

while the Kähler moduli only receive non-perturbative superpotential contributions [38].

1.2 Towards an effective description of warping

Steps towards including the effects of warping were taken in [29]. The simplest is to change the relation between the 10d Planck scale, the 4d Planck scale, and the volume of the internal manifold, to

$$\frac{M_{Pl,4}^2}{M_{Pl,10}^2} = \left(\frac{M_{Pl,10}}{2\pi}\right)^6 V_W,$$

where V_W is the "warped volume" of the internal manifold,

$$V_W = \int d^6y \sqrt{\tilde{g}_6}\, e^{-4A}\,. \tag{1.11}$$

Then, the potential and kinetic terms of the 4d effective action were computed by KK reduction. In particular, the kinetic terms for the moduli were obtained by varying the Einstein-Hilbert action around the metric ansatz Eq. (1.2), obtaining

$$G^W_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} K_W = \int d^6y \sqrt{\tilde{g}_6}\; e^{-4A} \partial_\alpha \tilde{g}_{mn}\, \partial_{\bar{\beta}} \tilde{g}^{mn}\,. \tag{1.12}$$

Then, substituting in the variations of the 6d metric with the moduli, it was found that the warp factor corrections change Eq. (1.8) to

$$K_W \stackrel{?}{=} -3\log(-i(\rho+\bar{\rho})) - \log\big(-i\int_M e^{-4A} J\wedge J\wedge J\big) - \log\big(-i\int_M e^{-4A}\Omega\wedge\bar{\Omega}\big) \tag{1.13}$$

However, in [26] and other works, it was realized that this analysis was somewhat oversimplified. One reason for this was that, in generalizing the KK ansatz to fields which vary in the four dimensions, one often needs to add terms which depend on derivatives of the fields. These "compensators" were discussed at some length in [26], with the conclusion that they are necessary, and do contribute to the kinetic terms, placing Eq. (1.13) in some doubt.

As mentioned before, KK modes becoming light in regions of strong warping also need to be included in the effective action. While there are a few compactifications in which a consistent truncation to a small subset of modes is possible [40], usually this is not the case, and there is no reason to believe it is so for an arbitrary Calabi-Yau compactification. Furthermore, in [26] it was claimed that mixings between the zero modes and their KK fluctuations are expected even at the level of the kinetic terms. Unlike the usual Calabi-Yau case, this would imply that the low energy dynamics truncated to metric moduli is inconsistent!

1.3 Some applications of warped effective theories

Warped compactifications are ubiquitous in the constructions of phenomenologically attractive string vacua, both for applications to particle physics and to cosmology. Thus, understanding effective theories in the presence of warping is essential for investigating the physics of such vacua where a hierarchy of scales can be generated. For example, in the construction of de-Sitter vacua in [38], the existence of a strongly warped region is a key ingredient

for the uplifting procedure. To properly describe the physics around these stabilized vacua, it is important to understand corrections to the effective theory due to warping. Furthermore, many crucial aspects of supersymmetry breaking in warped backgrounds – including its mediation mechanisms [41, 42], sequestering of supersymmetry breaking [43], and the computations of soft terms – depend on quantitative details of the warped effective theory. In fact, if warping is responsible for generating the electroweak hierarchy, a plausibly distinct signature of warped throats is the production of massive KK resonances at colliders; the precise signatures can depend on details of the warped geometry and of the interactions involving the KK modes [51].

On the other hand, explicit models of string inflation often make use of D-branes in warped throats since the warp factor can help in flattening the inflaton potential [44]. Predictions of these models depend strongly on the underlying effective action describing the closed string moduli as well as the moduli parametrizing the position of the mobile D-brane [45]. Hence, modifications to the effective action due to warping could have interesting phenomenological and model building consequenses. Furthermore, the end of D-brane inflation typically results in a brane and antibrane annihilating, with most of the energy ending up in KK modes of the warped throat [46, 47]. Reheating of the universe occurs when the KK modes decay on the branes which realize the Standard Model, and thus the details of reheating depend on the properties of the KK modes [46, 47, 48, 49, 50]. Therefore, the mixing among KK modes can have important consequences for these issues.

In light of these applications, it is an important problem to compute the dimensionally reduced effective action for warped compactifications, taking the previously identified challenges into account.

1.4 Summary of the paper

In this work we develop the theory of warped compactification as follows. In section 2 the theory is analyzed from a ten dimensional perspective in the axial gauge $g_{\mu m} = 0$. We identify the 10d fluctuations and compute their equations of motion. In subsection 2.1 we define, for each type of fluctuation, a basis of "warped" internal wavefunctions which will be used throughout the work. An important result is that metric fluctuations must satisfy the constraints Eqs. (2.21) and (2.22), which follow from the $(\mu\nu)$ and (μm) Einstein equations. We provide a general formula for the effective action in Eq.(2.24).

Section 3 presents the four dimensional kinetic terms for the different fluctuations. It is argued that there are no terms with two space-time derivatives mixing the metric moduli with any of the other light modes, in the

basis of “warped” KK modes defined in subsection 2.1. Therefore, even in the strongly warped limit it is consistent to study the propagators associated to such moduli independently of the other fields. In Eq. (3.21) we present the warped Kähler potential for complex moduli and address various puzzles related to this term.

In section 4 we study the geometrical KK masses, nonvanishing even for zero fluxes. Subsection 4.1 provides a mathematical and physical justification for the use of the previous basis of “warped” eigenmodes. We show that these mass terms do not mix the metric moduli with KK excitations, while we do find quadratic couplings between massive graviton and internal metric modes. This suggests that a certain geometrical Higgs mechanism may be at work in the warped compactifications being analyzed.

Finally, in section 5 we find the flux potential for the metric fluctuations, including KK modes. The result, given in Eq. (5.6), is a warped generalization of the Gukov-Vafa-Witten potential. Interestingly, this potential exhibits possible mixings between the moduli and their KK tower. Our analysis applies to nontrivial axio-dilaton fluctuations as well, which also mix with the metric moduli. In the appendices we collect various useful formulas and show some of the computations in detail.

2 From 10 to 4: Warped Fluctuations and Effective Action

In this section, the general procedure for obtaining the effective action for warped flux compactifications will be described. First, we identify the ten dimensional fluctuations of the metric and matter which give rise to 4d fields. Key to our analysis is the choice of a supergravity gauge fixing condition, which simplifies the problem considerably. Examining the equations of motion, we find the existence of *constraint equations*; these relate different fluctuations to each other and fix residual gauge transformations. Finally, we provide a general formula for computing the effective action.

2.1 10d perspective for fluctuations in IIB supergravity

From a ten-dimensional point of view, the dynamics follows by considering infinitesimal fluctuations around the previous backgrounds in which the moduli are spacetime dependent, and then solving the corresponding equations of motion. The zero mode sector includes the complex and Kähler moduli $u^I = (\rho^i, S^\alpha)$, the 4d graviton $h_{\mu\nu}(x)$, the axio-dilaton $\tau_0(x)$ (both are constant on the internal manifold), and the various massless p-form fields coming

from decomposing (C_2, B_2, C_4) into harmonic forms. For each of them we will include the corresponding tower of KK excitations.

We will take the fluctuations of the 10-dimensional metric, in the presence of dynamical moduli, to have the form

$$\delta(ds^2) = \delta g_{\mu\nu} dx^\mu dx^\nu + \delta g_{mn} dy^m dy^n \,. \tag{2.1}$$

where

$$\delta g_{\mu\nu}(x,y) = e^{2A(y)}[2\,\delta A(x,y)\eta_{\mu\nu} + \delta_K g_{\mu\nu}(x,y)]\,, \tag{2.2}$$

$$\delta g_{mn}(x,y) = e^{-2A(y)}[-2\,\delta A(x,y)\tilde{g}_{mn}(y) + \delta\tilde{g}_{mn}(x,y)]\,. \tag{2.3}$$

Here $\delta_K g_{\mu\nu}$ are 4d graviton KK modes (which are not necessarily transverse-traceless), while $\delta\tilde{g}_{mn}$ encode the metric moduli u^I and their KK modes. Since the warp factor depends on the moduli, a fluctuation $\delta\tilde{g}_{mn}$ induces in turn a variation $\delta A = u^I \partial A/\partial u^I$. It is not consistent to set $\delta A = 0$.

In general there are also off-diagonal moduli-dependent fluctuations of the form $\delta g_{\mu m} \sim \partial_\mu u(x) B_m(y)$ [26, 39]. The reason for this particular form is that in the limit in which the moduli become constant, we should recover the background (1.2). These are gauge dependent and as we will discuss elsewhere [62] we can remove them by going to an "axial" type gauge

$$\delta g_{\mu m} = 0\,. \tag{2.4}$$

This choice will make the four dimensional degrees of freedom manifest. As explained in the next subsection, the remaining diffeomorphisms that preserve this condition are fixed by the equations of motion.

To isolate the zero modes from their KK partners, we expand the metric fields in a basis of eigenmodes for the internal manifold:

$$\delta_K g_{\mu\nu}(x,y) = \sum_{I_1} h^{I_1}_{\mu\nu}(x) Y^{I_1}(y)\,. \tag{2.5}$$

$$\delta\tilde{g}_{mn}(x,y) = \sum_{I_2} u^{I_2}(x)\, Y^{I_2}_{mn}(y)\,, \tag{2.6}$$

The multi-index I_i, $i = 1, 2$, runs over the different types of metric fluctuations and, for each type of fluctuation, over the 4d KK tower. $I_i = 0$ gives the zero mode of the appropriate Laplacian on the internal manifold. To be precise, in Eq. (2.6) we must specify a gauge. This will be Eq. (2.22) as we discuss below.

We certainly have the freedom to choose different complete bases of internal wavefunctions; this amounts to making field redefinitions in the four-dimensional theory. Different choices represent the extra dimensions in rather

different ways and our aim is to find the one that yields the simplest description. For instance, for a scalar mode in six dimensions, there are two natural possibilities, either

$$\tilde{\nabla}^2 Y_i(y) = e^{-4A(y)} \lambda_i^2 Y_i(y) , \tag{2.7}$$

or the usual unwarped KK modes,

$$\tilde{\nabla}^2 \mathcal{Y}_i(y) = \nu_i^2 \mathcal{Y}_i(y) . \tag{2.8}$$

However, since in a warped compactification the mass eigenmodes will be given by $Y_i(y)$, it is much simpler to expand the ten dimensional fields in the first basis of "warped" KK modes.

One can check that Eq. (2.7) is a well-defined Sturm-Liouville problem; thus if we use the inner product in which this linear operator is self-adjoint, non-degenerate eigenvectors will be orthogonal and we can orthogonalize degenerate eigenvectors. Thus we can choose a basis in which

$$\int d^6y \sqrt{\tilde{g}_6}\, e^{-4A}\, Y_i(y) Y_j(y) = G\, \delta_{ij} . \tag{2.9}$$

In the following, this is made explicit in each sector.

Metric fluctuations

We will take the eigenmodes (2.5), (2.6) to be solutions to the respective eigenvalue equations,

$$\tilde{\nabla}^2 Y^{I_1} = e^{-4A(y)} \lambda_{I_1}^2 Y^{I_1} , \tag{2.10}$$

$$\frac{1}{2} \tilde{\Delta}_L Y_{mn}^{I_2}(y) = \delta \tilde{G}_{mn} = e^{-4A(y)} \lambda_{I_2}^2 Y_{mn}^{I_2}(y) ; \tag{2.11}$$

here $\tilde{\Delta}_L$ is the Lichnerowicz laplacian for $\tilde{g}_{mn}$. The eigenmode expansions (2.11, 2.10) lead to orthogonality relations between different modes

$$\frac{1}{2} \int d^6y \sqrt{\tilde{g}_6} e^{-4A} Y^{I_1} Y^{J_1} = \mathcal{M}^{kk} \delta_{I_1,J_1} . \tag{2.12}$$

$$\int d^6y \sqrt{\tilde{g}_6}\, e^{-4A(y)}\, Y_{mn}^{I_2}(y) \bar{Y}^{J_2\, \widetilde{mn}}(y) = G^{(u)}\, \delta_{I_2,J_2} . \tag{2.13}$$

Indices with tildes are raised with $\tilde{g}^{mn}$.

Dilaton fluctuations

Fluctuations of the dilaton are of the form $\tau = \tau_0 + \delta\tau(x,y)$, where $\mathrm{Im}\, \tau_0 = g_s^{-1} \gg 1$ and is constant in the internal space. Since the axio-dilaton is a scalar from the six-dimensional point of view, its expansion in KK modes is the same as for the graviton:

$$\delta\tau = \delta\tau_0(x) + \delta_K \tau(x,y) = \delta\tau_0(x) + \sum_{I_1} t^{I_1}(x) Y^{I_1}(y) . \tag{2.14}$$

The orthogonality relation for the dilaton KK modes then reads,

$$-\frac{1}{4(\mathrm{Im}\tau_0)^2}\int d^6y\sqrt{\tilde{g}_6}\, e^{-4A}\, Y^{I_1}(y)Y^{J_1}(y) \equiv \mathcal{M}_\tau^{kk}\,\delta_{I_1 J_1}\,. \tag{2.15}$$

p-form fluctuations

The various antisymmetric tensors have an expansion of the form

$$\begin{aligned} &F_3 \to F_3 + d\,\delta C_2\ ,\ H_3 \to H_3 + d\,\delta B_2\ ,\\ &F_5 \to F_5 + d\,(\delta\alpha)\wedge d^4x + d\,\delta C_4\,. \end{aligned} \tag{2.16}$$

(F_3, H_3, F_5) denote the background GKP values, while the derivative variations include the zero modes (harmonic forms) and their KK excitations. Note that because of the background BPS relation $\alpha = e^{4A}$, fluctuations of the 4-form are induced by fluctuations of the warp factor (which are in turn induced by fluctuations of moduli), e.g. $\delta\alpha = \delta e^{4A}$.

As with the metric, when the moduli are promoted to spacetime dependent fields in general one should include terms in the p-form fluctuations proportional to spacetime derivatives of the moduli, $\delta C_2 \sim du^I \wedge T_I$, $\delta C_4 \sim du^I \wedge S_I^{(3)} + ...$ As shown in Appendix A, solving the flux equations of motion leads to the identification of these fluctuations as pure gauge, and we will fix the gauge freedom by choosing them to vanish. This is the analog of Eq. (2.4). This is a crucial simplification and is particular to GKP backgrounds. In more general settings [26], the previous is not true and the ten dimensional analysis becomes quite involved.

In principle, a nontrivial warp factor could induce mixings between the four dimensional scalars coming from these p-forms and the metric moduli. However, we will argue in section 5.1 that this is not the case; for this reason, we will not perform a full analysis of this sector.

2.2 Ten dimensional fluctuated equations

The linearized equations for the fluctuations are

$$\delta G_{MN} = \kappa_{10}^2\delta T_{MN}\ ,\ d\delta F_5 = \delta(H_3\wedge F_3) + 2\kappa_{10}^2\delta(T_3\rho_3^{loc})\,, \tag{2.17}$$

$$d\,\delta\big[e^{4A}(\star_6 G_3 - iG_3)\big] = 0\,,\ \delta(\star_{10}F_5) = \delta F_5\,, \tag{2.18}$$

$$\nabla_M\nabla^M\delta_K\tau = -\frac{i}{12}\delta(G_3\cdot G_3)\,, \tag{2.19}$$

using the usual formula

$$\delta R_{MN} = -\frac{1}{2}\nabla^P\nabla_P\delta g_{MN} - \frac{1}{2}\nabla_M\nabla_N g^{PQ}\delta g_{PQ} + \frac{1}{2}\nabla^P\nabla_M\delta g_{NP} + \frac{1}{2}\nabla^P\nabla_N\delta g_{MP}\,. \tag{2.20}$$

These equations are discussed in more detail in the Appendix.

An important point of this work is to recognize that some of these equations contain at most first order space-time derivatives. These "constraints" are familiar from systems with gauge redundancies. For example, in general relativity, $G_{0i} = T_{0i}$ does not contain second order time derivatives, and so it has to be satisfied by any consistent solution, at all times [53]. Another famous example of this type is Gauss' law in electrodynamics. We now identify such initial value constraints for warped supergravities. They will play a crucial role in simplifying the four dimensional action.

One such constraint comes from the $(\mu\nu)$ Einstein equation[2],

$$\delta G_{\mu\nu} - \kappa_{10}^2 \delta T_{\mu\nu} = \partial_\mu \partial_\nu (4\delta A - \frac{1}{2}\delta\tilde{g}) + \eta_{\mu\nu}[...] = 0\,.$$

Subtracting out the trace of this equation, we obtain the constraint

$$\delta A = \frac{1}{8}\delta\tilde{g}\,. \tag{2.21}$$

Computationally, this is a very useful relation, allowing us to replace the potentially complicated fluctuation δA by $\delta\tilde{g}$. This equation has a nice physical interpretation as the invariance of the warped volume Eq. (1.11) with respect to fluctuations of the moduli, $\delta V_W = 0$.

Another constraint comes from the (μm) Einstein equation

$$\delta G_{\mu m} - \kappa_{10}^2 \delta T_{\mu m} = \frac{1}{2}\partial_\mu \left[\tilde{\nabla}^n(\delta\tilde{g}_{mn} - \frac{1}{2}\tilde{g}_{mn}\delta\tilde{g}) - 4\partial^{\tilde{n}} A\, \delta\tilde{g}_{mn}\right] = 0\,,$$

which indicates that the condition

$$\tilde{\nabla}^n\big(\delta\tilde{g}_{mn} - \frac{1}{2}\tilde{g}_{mn}\,\delta\tilde{g}\big) = 4(\partial^{\tilde{n}} A)\,\delta\tilde{g}_{mn} \tag{2.22}$$

must be satisified. This resembles a warped generalization of the harmonic gauge and, indeed, it fixes the remaining six dimensional diffeomorphisms that leave the axial gauge (2.4) invariant. However, it must be stressed that this is not a choice, but a constraint imposed by the dynamics. Fluctuations that do not satisfy this consistency expression will not give a consistent ten dimensional solution.

Finally, another set of constraints follows from the moduli-dependent parts of the five form and three form equations of motion, self duality, and Bianchi identities, (2.17-2.18). We will not write these out in detail here – explicit use of them will be made in the Appendix.

[2] A similar expression is found in the dimensional reduction of the radion in the Randall-Sundrum model [54].

2.3 Four dimensional effective action

As the final step in our general discussion, we wish to understand the dynamics from a four dimensional action principle, by first compactifying the supergravity action on a background satisfying the ten-dimensional equations of motion, and then integrating over the internal coordinates, along the lines of [55].

There are some known subtleties in doing this. First, the type IIB supergravity action is ill-defined due to the self-duality of the five form. The procedure which will be followed here is to project out half of the 4-dimensional degrees of freedom of the five form and double the coefficients of the F_5^2 and Chern-Simons term [29]. Second, the Gibbons-Hawking-York term [56] must be included in the dimensional reduction to cancel certain total derivative terms of the variation of the gravitational action.

The dimensionally reduced effective action is then obtained by expanding

$$S = \int_M d^{10}x\sqrt{g}\,(g^{MN}\,R_{MN} + \mathcal{L}_{matter}) + 2\int_{\partial M} d^9x\,\sqrt{h}\,K \tag{2.23}$$

to second order in fluctuations. After some algebra, the result is,

$$S_{eff} = \frac{1}{4\kappa_{10}^2}\int d^{10}x\,\sqrt{g}\,\Big[-(\delta g)^{MN}\big(\delta G_{MN} - \delta T_{MN}\big) + \delta^2\mathcal{L}_{matter}\Big] + \mathcal{O}(\delta g^3)\,, \tag{2.24}$$

where $\delta^2\mathcal{L}_{matter}$ represents second order fluctuations of the flux and dilaton terms in the effective action with respect to the flux and dilaton fields. This reproduces the correct ten dimensional equations of motion. To perform the dimensional reduction, one expands the fluctuations in terms of internal eigenmodes and integrates over the compactification space, while imposing the constraint equations (2.21, 2.22) derived in the previous subsection.

Formally, in this process we are including all the (infinite tower of) KK modes, to quadratic order. The details of each particular region of strong warping will then determine a truncation of the KK tower to keep only the lightest modes. Higher order interactions may also be analyzed by further expanding, for instance, the Einstein-Hilbert term. The existence of trilinear couplings between moduli and KK modes may have interesting consequences. In the rest of the paper we will study this effective action, sector by sector, in detail. We present the full computation in Appendix B.

3 Warped Kinetic Terms and Moduli Space Metrics

This section is devoted to the analysis of the terms in the effective action that contain space-time derivatives. The main result will be that the kinetic terms for all sectors decouple and are diagonal in the KK mode expansion,

$$\mathcal{L}_{kin} = G^{(u)} \sum_{I_2} u^{I_2} \Box \bar{u}^{I_2} + \mathcal{M}_h^{kk} \sum_{I_1} E_{I_1}^{\mu\nu} h_{\mu\nu}^{I_1} + \mathcal{M}_\tau^{kk} \sum_{I_1} t^{I_1} \Box \bar{t}^{I_1} .$$

Here, $E_{\mu\nu}$ is the linearized four dimensional Einstein tensor and I_i runs over the different moduli and their KK towers. This implies that (even with a nontrivial warp factor) the propagators for the moduli and for the light KK modes do not mix. The diagonality of the kinetic terms implies that the Kähler potential is also diagonal to quadratic order in the fluctuations of the moduli and the dilaton (and their KK modes),

$$K = G^{(u)} \sum_{I_2} u^{I_2} \bar{u}^{I_2} + \mathcal{M}_\tau^{kk} \sum_{I_1} t^{I_1} \bar{t}^{I_1} . \tag{3.1}$$

3.1 Axio-dilaton and p-form modes

As a warm-up, we first consider the kinetic terms for the axio-dilaton and the p-forms. Here the dimensional reduction is simple but nonetheless ilustrates some of the features that will appear in the more involved analysis of metric fluctuations.

From Eq. (1.1), the axio-dilaton is a ten-dimensional scalar field with a nonlinear metric. Expanding around the background value τ_0 according to (2.14), the kinetic term turns out to be

$$\mathcal{L}_{kin}^{(\tau)} = \sum_{I_1, J_1} G_{I_1 J_1}^{(\tau)} t^{I_1}(x) \Box \bar{t}^{J_1}(x) \tag{3.2}$$

where the warped metric is

$$G_{I_1 J_1}^{(\tau)} = \frac{1}{4 (\mathrm{Im}\, \tau_0)^2 V_W} \int d^6 y \sqrt{\tilde{g}_6}\, e^{-4A(y)} Y_{I_1}(y) Y_{J_1}(y) . \tag{3.3}$$

Notice that the dilaton metric (3.3) is proportional to the orthogonality relation on the internal space (2.15), so that the Kähler potential, to quadratic order, becomes (taking $\mathcal{M}_\tau^{kk} \equiv G_{I_1 I_1}^{(\tau)}$)

$$K^{(\tau)} = \mathcal{M}_\tau^{kk} \sum_{I_1} t^{I_1}(x) \bar{t}^{I_1}(x) . \tag{3.4}$$

The conclusion is that, at this order, there is no mixing between the various dilaton KK modes in the kinetic term. Had we used the unwarped KK expansion of Eq. (2.8), the kinetic term would have exhibited complicated mixings.

Next we discuss a rather different behavior, arising from the one-form KK modes of C_4:

$$\delta C_4 = \sum_I V_\mu^I(x) dx^\mu \wedge \chi^I(y)\,,$$

where $\chi^I(y) = \frac{1}{3!}\chi^I_{mnp}\, dy^{mnp}$. This case is relevant for D-term supersymmetry breaking [57]. Replacing this in the $\tilde{F}_5^2$ term of the action (1.1), we find

$$\mathcal{L}^{(V)}_{kin} = G^{(V)}_{IJ}\, F^I \wedge \star_4 F^J\,. \tag{3.5}$$

The metric appearing here is

$$G^{(V)}_{IJ} = \sum_{I,J} \frac{1}{4\,V_W} \int_M \chi_I(y) \wedge \star_6 \chi_J(y)\,, \tag{3.6}$$

Therefore, the field space metric coincides with the unwarped one, with the result that the corresponding D-term cannot be made parametrically small by the large hierarchy of the throat. The field space metric $G^{(V)}$ for the massless mode can be shown to coincide with $\mathrm{Im}\,\partial^2\mathcal{F}$, where $\mathcal{F}$ is the Calabi-Yau prepotential.

3.2 Graviton fluctuations

After having gained some intuition with the previous simpler sectors, we will now consider the metric fluctuations,

$$\delta(ds^2) = e^{2A}\big[2\delta A\, \eta_{\mu\nu} + \delta_K g_{\mu\nu}\big] dx^\mu dx^\nu + e^{-2A}\big[-2\delta A\, \tilde{g}_{mn} + \delta\tilde{g}_{mn}\big] dy^m dy^n\,. \tag{3.7}$$

The kinetic term for this sector follows from dimensionally reducing the Einstein-Hilbert part of the supergravity action, according to the prescription Eq. (2.24).

The variation of the warp factor makes this computation highly nontrivial in at least two aspects. First, from the space-time variation we expect mixings between the trace part of the graviton mode[3] and the moduli through δA, $\delta\tilde{g}$. Further, the internal metric variation is no longer proportional to $\delta\tilde{g}_{mn}$ (since it includes δA fluctuations) so the relation between a complex

[3]Note that we have not chosen the standard transverse traceless gauge for the graviton, which is in general not consistent with axial gauge [26].

modulus of the underlying Calabi-Yau ($\delta\tilde{G}_{mn}=0$) and a zero mode of the full warped metric ($\delta G_{MN}=0$) may be very involved.

Our first result will be to show that, in spite of the possible couplings suggested by (3.7), there are no space-time derivative mixings between $\delta_K g$ and $\delta\tilde{g}_{mn}$. The simplest way of understanding this is by doing a conformal transformation, and for this it is actually better to work with the metric containing the fluctuations to all orders:

$$ds^2 = e^{2A(x,y)}\, g_{\mu\nu}(x,y)\, dx^\mu dx^\nu + e^{-2A(x,y)}\, \tilde{g}_{mn}(x,y)\, dy^m dy^n\,. \tag{3.8}$$

The x dependence in A and $\tilde{g}_{mn}$ comes from promoting the moduli to space-time dependent fields and from their KK modes. The graviton is associated to the fluctuating metric $g_{\mu\nu}(x,y)$ and does not induce warp factor fluctuations.

Consider the conformal transformation

$$ds^2 = e^{2A(x,y)}\, d\hat{s}^2\ ,\ \hat{g}_{\mu\nu}=g_{\mu\nu}\ ,\ \hat{g}_{mn}=e^{-4A}\,\tilde{g}_{mn}\,, \tag{3.9}$$

which leads to a change in the Ricci scalar [53]

$$R\to e^{-2A}\big(\hat{R}+9\times 8\,\hat{g}^{LM}\partial_L A\,\partial_M A\big)\,, \tag{3.10}$$

after an integration by parts. The conformally rescaled metric $\hat{g}_{MN}$ does not mix the KK graviton and warp factor, making the decoupling of these fluctuations manifest. Dimensionally reducing $\hat{R}$ on this ansatz for $\hat{g}_{MN}$ leads to kinetic terms for the graviton and internal metric fluctuations without off-diagonal mixings. Further, the spacetime derivative contribution of the extra term in (3.10) does not contain graviton pieces (at quadratic order). This proves that there are no space-time derivative mixings. This result is rederived in Appendix B, by performing the computation in the original unrescaled metric.

After having established this, it is straightforward to compute the kinetic term for the graviton modes, since we only need to consider a metric perturbation

$$\delta(ds^2) = e^{2A(y)}\,\delta_K g_{\mu\nu}(x,y)dx^\mu dx^\nu\,. \tag{3.11}$$

The result is a warped version of the linearized Einstein-Hilbert action around a flat background [53],

$$\begin{aligned} S^{(h)}_{kin} &= \frac{1}{2\kappa_4^2 V_W}\int d^4x\int d^6y\sqrt{\tilde{g}_6}e^{-4A}\delta_K g^{\mu\nu}\,\delta_K G^{(4)}_{\mu\nu}\\ &= \frac{1}{2\kappa_4^2 V_W}\int d^4x\int d^6y\sqrt{\tilde{g}_6}e^{-4A}\Big[-\frac{1}{2}(\delta_K g^{\mu\nu}\,\Box\delta_K g_{\mu\nu}-\delta_K g\,\Box\delta_K g)+\\ &+ \delta_K g^{\mu\nu}(\partial^\sigma\partial_{(\mu}\delta_K g_{\nu)\sigma}-\partial_\mu\partial_\nu\delta_K g)\Big]\,. \end{aligned} \tag{3.12}$$

Indices are raised with $\eta^{\mu\nu}$, and $\delta_K g := \eta^{\mu\nu}\delta_K g_{\mu\nu}$. In harmonic gauge the last line vanishes and we recover the usual kinetic term; however, at this point we prefer to keep the graviton gauge arbitrary.

Expanding $\delta_K g_{\mu\nu}$ in the internal fluctuations of (2.5), the field space metric is seen to be

$$G^{(h)}_{I_1 J_1} = \frac{1}{4\,V_W} \int d^6y \sqrt{\tilde{g}_6}\, e^{-4A(y)} Y_{I_1}(y) Y_{J_1}(y)\,, \tag{3.13}$$

which, from the orthogonality relation (2.12), is proportional to the identity matrix. We thus obtain a warped generalization of the usual gravity lagrangian (again taking $\mathcal{M}^{kk}$ to be the diagonal part of the metric (3.13))

$$\mathcal{L}^{(h)}_{kin} = \mathcal{M}^{kk} \sum_{I_1} E^{I_1}_{\mu\nu}(x)\, h^{\mu\nu}_{I_1}(x) \tag{3.14}$$

in terms of the linearized Einstein tensor

$$E^{I_1}_{\mu\nu}(x) := \frac{1}{2}\big(\Box h^{I_1}_{\mu\nu} - \eta_{\mu\nu}\Box h^{I_1} + \partial_\mu\partial_\nu h^{I_1} - \partial_\mu\partial^\lambda h^{I_1}_{\lambda\nu} - \partial_\nu\partial^\lambda h^{I_1}_{\lambda\mu} + \eta_{\mu\nu}\partial^\lambda\partial^\rho h^{I_1}_{\lambda\rho}\big)\,. \tag{3.15}$$

Since the four dimensional theory has $\mathcal{N} = 1$ supersymmetry, this is the bosonic part of a D-term. In terms of the real vector superfields H_μ (which contains the graviton and gravitino) and E_μ (whose $\bar{\theta}\theta$ component is the Einstein tensor), this D-term is

$$\mathcal{L}^{(h)}_D = M^{kk} \sum_{I_1} \eta^{\mu\nu} E^{I_1}_\mu\, H^{I_1}_\nu\,, \tag{3.16}$$

where we follow the notations of [58].

3.3 Kähler potential for internal metric fluctuations

At last, we are ready to address the problem of computing the Kähler potential for the internal metric fluctuations. From our previous considerations, this comes from the second order fluctuation of $\int R_{(10)}$ generated by the complex structure and Kähler moduli metric perturbations,

$$\delta(ds^2) = 2e^{2A}\,\delta A\,\eta^{\mu\nu}dx^\mu dx^\nu + e^{-2A}\big(-2\delta A\,\tilde{g}_{mn} + \delta\tilde{g}_{mn}\big)dy^m dy^n\,. \tag{3.17}$$

The universal Kähler modulus presents additonal subtleties, and its analysis is left for future work.

The full computation is quite long, so we relegate it to the Appendix. As a summary, the kinetic terms come from

$$-\frac{1}{2}\int d^4x\, d^6y\, \sqrt{\tilde{g}_6} e^{-2A}\,(\delta g)^{mn}\delta G_{mn}\,.$$

This turns out to be a sum of various pieces containing $\delta\tilde{g}_{mn}$, $\delta\tilde{g}$ and δA. After using the constraint Eq. (2.21), surprisingly most of the terms cancel and we are left with

$$S_{kin}^{(u)} = \frac{1}{8\kappa_{10}^2}\int d^4x \int d^6y\,\sqrt{\tilde{g}_6}\,e^{-4A}\,\delta\tilde{g}^{mn}\Box\,\delta\tilde{g}_{mn} = \frac{1}{2\kappa_4^2}\int d^4x\,G^{(u)}_{I_2\bar{J}_2}\,u^{I_2}\,\Box\bar{u}^{J_2} \tag{3.18}$$

where we have used the expansion of Eq. (2.6) to write this in terms of a field space metric,

$$G^{(u)}_{I_2\bar{J}_2} = \frac{1}{4V_W}\int d^6y\,\sqrt{\tilde{g}_6}\,e^{-4A}\,Y_{I_2,\,mn}(y)\bar{Y}^{\widetilde{mn}}_{J_2}(y)\,. \tag{3.19}$$

Note that these expressions are not invariant under 6d gauge transformations $\delta\tilde{g}_{mn} = \nabla_{(m}\xi_{n)}$, and thus to make them well defined we need to specify the gauge, which is Eq. (2.22). We will discuss this point further in [62].

Again, the metric (3.19) is proportional to the orthogonality relation (2.13), so that the kinetic term is diagonal in the different KK levels,

$$\mathcal{L}_{kin}^{(u)} = G^{(u)}\sum_{I_2} u^{I_2}\Box\bar{u}^{I_2}\,. \tag{3.20}$$

This is one of the main results of our work. If one is only interested in the propagator of the metric moduli, then even at strong warping it is consistent to truncate the analysis to the zero KK level. As was pointed out by [26], a warp factor does induce mixings between the moduli and the *unwarped* KK modes, given in Eq. (2.8). We will argue in section 4 that such modes don't represent light four dimensional excitations. Rather, these are given by the warped eigenvectors of Eq. (2.11), in terms of which there is no kinetic mixing.

Now we can understand better the construction of the complex moduli metric. From Eq. (3.19), the field space metric for complex deformations reads

$$G^{(S)}_{\alpha\bar{\beta}} = \frac{1}{4V_W}\int d^6y\,\sqrt{\tilde{g}_6}\,e^{-4A}\,\delta_\alpha\tilde{g}_{mn}\,\delta_{\bar{\beta}}\tilde{g}^{mn}\,, \tag{3.21}$$

and we have changed to a more familiar notation where $\delta_\alpha\tilde{g}_{mn}$ denotes the wavefunction $Y^{I_2}_{mn}$ associated to a complex structure deformation S^α. This appears at first sight to agree with the form conjectured by [29], but there are in fact some crucial differences due to the consistency of the dimensional reduction that one should take into account. The analysis is based on the gauge choice

$$\delta_\alpha g_{\mu m} = 0 \tag{3.22}$$

and the metric fluctuations must satisfy the constraints

$$\delta_\alpha A = \frac{1}{8}\delta_\alpha \tilde{g}\ ,\ \tilde{\nabla}^n\big(\delta_\alpha \tilde{g}_{mn} - \frac{1}{2}\tilde{g}_{mn}\,\delta_\alpha \tilde{g}\big) = 4(\partial^{\tilde{n}} A)\,\delta_\alpha \tilde{g}_{mn}\,, \tag{3.23}$$

which follow from the ten dimensional equations $\delta_\alpha G_{\mu\nu} = \delta_\alpha G_{\mu m} = 0$. Eqs. (3.22) and (3.23) specify a unique representative $\delta_\alpha \tilde{g}_{mn}$ from each class of (diffeomorphism) equivalent metric fluctuations. Therefore (3.21) gives a well-defined result.

To see how (3.21) differs from the standard moduli space metric, as in [55] we write the metric fluctuation in terms of the (2, 1) form χ which is harmonic in the *unwarped* metric,

$$\frac{\partial \tilde{g}_{mn}}{\partial S^\alpha} = -\frac{1}{||\Omega||^2}\bar{\Omega}_m{}^{rs}\chi_{\alpha,\,rsn}\,. \tag{3.24}$$

Then

$$\delta_\alpha \tilde{g}_{mn} = \frac{\partial \tilde{g}_{mn}}{\partial S^\alpha} + \delta_\alpha \tilde{g}^*_{mn} \tag{3.25}$$

where $\delta_\alpha \tilde{g}^*_{mn}$ is determined by the constraint equations (3.23).

One can check that given a general form for the warp factor, Eq. (3.23) requires $\delta_\alpha \tilde{g}^*_{mn} \neq 0$, so this extra term leads to further warp corrections in the metric Eq. (3.21),

$$G^{(S)}_{\alpha\bar{\beta}} = \frac{1}{4V_W}\int e^{-4A}\chi_\alpha \wedge \bar{\chi}_{\bar{\beta}} + \frac{1}{4V_W}\int d^6y\,\sqrt{\tilde{g}_6}\,e^{-4A}\frac{\partial \tilde{g}_{mn}}{\partial S^\alpha}\,\delta_{\bar{\beta}}\tilde{g}^{*mn} + ... \tag{3.26}$$

The upshot is that, unless there are further conspiracies in the determination of the warp factor which cause the extra terms to cancel, the metric on complex structure moduli space is not Eq. (1.12) found in [29] but instead contains extra terms. It would be interesting to compare this result with the moduli space kinetic term in [28], which was obtained by generalized complex geometry methods, and to the warping corrections suggested by a different type of analysis for the universal Kähler modulus sector in [34]. We will return to this question in [62].

3.4 Supersymmetry considerations

We conclude by interpreting the previous results from the point of view of the unbroken $\mathcal{N} = 1$ supersymmetry.

In the unwarped case, the massless four dimensional spectrum falls into the following $\mathcal{N} = 2$ multiplets [52]: one gravity multiplet with matter fields $(g_{\mu\nu}, V^0)$, $h^{(2,1)}$ vector multiplets (V^α, S^α), $h^{(1,1)}$ hypermultiplets (v^A, b^A, c^A, ρ_A),

$\mathcal{N}=1$ multiplet	multiplicity	matter content	$\exp(-4nA)$
gravity	1	$g_{\mu\nu}$	$n=1$
vector	$1+h^{(2,1)}$	V^K	$n=0$
chiral	$1+h^{(2,1)}$	$(\tau,\, S^\alpha)$	$n=1$
chiral	$h^{(1,1)}$	(ρ^A, v^A)	$n=1$
chiral	$h^{(1,1)}$	(b^A, c^A)	$n=0$
tensor	$h^{(1,1)}$	(B_2, C_2)	$n=2$

Table 1: Supermultiplet structure of type IIB supergravity compactified on a warped Calabi-Yau. The warp factor power n refers to the dependence of the field space metric on e^{-4A}.

and a tensor multiplet (B_2, C_2, τ). Here, V^K are space-time gauge fields that come from decomposing C_4 in harmonic 3-forms; S^α and ρ_A are, respectively, the complex and Kähler moduli. The rest of the fields come from expanding the ten-dimensional 2- and 4-forms as zero forms (B_2, C_2) and 2-forms (b^A, c^A) on the internal part.

In the presence of warping, the four dimensional massless spectrum is shown in Table 1. The typical warped field space metric is of the form

$$G_{ij} = \frac{1}{V_W} \int d^6y \sqrt{\tilde{g}_6}\, e^{-4nA(y)}\, \omega_i(y)\omega_j(y) \tag{3.27}$$

where $\omega_i(y)$ is the wavefunction on the internal space, and the n-dependence is given in the table.

Massive supermultiplets associated to the KK modes also need to be included. While the supersymmetry description of spin 1 and 2 massive multiplets is more involved, in our case they can be obtained from their massless counterparts, via the appropriate super Higgs mechanism. The outcome is that each KK level exhibits supermultiplets analogous to the ones in Table 1, and the kinetic terms in the massless and massive case agree. As a consequence, we can use the same arguments as in the massless case to restrict the possible derivative terms.

In this way we can understand why, for instance, there are no kinetic mixings between the internal metric fluctuations (chiral superfields) and the KK gravitons (real vector superfields). Similarly, the kinetic term for the gauge supermultiplet is an F-term of the spinor superfield W_α. This forbids any warp correction in such a term, because e^{-4A} is not holomorphic.

4 Geometric Masses for KK modes

In this section we now compute the geometric masses (nonvanishing in the limit $G_3 \to 0$) for the various KK excitations. Flux-induced mass terms are discussed in section 5. It will be seen that such terms do not induce additional mixings between the zero modes and their KK excitations, precisely due to the orthogonality relations. We do find mixings between graviton KK modes and those from the internal metric, and point out that they may be interpreted as a warped generalization of a Higgs-type mechanism.

4.1 Scalar field case

To illustrate the physics behind the choice of the proper mode expansion, we now work in detail the case of a ten-dimensional scalar field; the other modes follow a similar pattern. Consider the action with a possible nontrivial potential,

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left(\partial_M \phi \, \partial^M \phi + V(\phi) \right) . \tag{4.1}$$

Using the ansatz

$$\phi(x, y) = \sum_i \varphi_i(x) Y_i(y)$$

the dimensionally reduced action becomes

$$S = -\frac{1}{2\kappa_{10}^2} \int d^4x \Big[(Y_i, e^{-4A} Y_j) \, \varphi_i \Box \varphi_j + (Y_i, \tilde{\nabla}^2 Y_j) \, \varphi_i \varphi_j + V(\varphi) \Big] \tag{4.2}$$

where we have introduced the natural inner product on the Calabi-Yau manifold,

$$(f, g) := \int d^6 y \sqrt{\tilde{g}_6} \, f(y) g(y) \, . \tag{4.3}$$

Both operators e^{-4A} and $\tilde{\nabla}^2$ are self-adjoint with respect to this product, so that we have a well-defined action.

A preferred basis for $Y_i(y)$ would be the one in which both the field space metric and mass matrix are simultaneously diagonalized, if possible. In our case, such functions are given as the eigenvectors of the following differential problem:

$$\tilde{\nabla}^2 \, Y_i(y) = e^{-4A(y)} \, \lambda_i^2 \, Y_i(y) \, . \tag{4.4}$$

Then the action acquires the desired diagonal form

$$S = -\frac{1}{2\kappa_{10}^2} \int d^4x \Big[G \, \varphi_i (\Box + \lambda_i^2) \varphi_i + V(\varphi) \Big] . \tag{4.5}$$

One arrives to the same results by requiring that the 4d scalar has a well-defined mass, $\eta^{\mu\nu}\partial_\mu\partial_\nu\varphi_i = -\lambda_i^2\varphi_i$. These are the mass eigenstates in the limit $V \to 0$. It turns out that Eq. (4.4) has a nice interpretation as a Schrödinger equation for the wavefunction Y_i with a potential determined by the warp factor [33, 49]. Light warped KK modes correspond to the bound states of such potential, while the unwarped modes are associated to states whose interactions are warp factor insensitive in a box of size V_W. The low energy dynamics contains massless modes (such as the 4d graviton) and these bound states. One could insist on describing the system with the unwarped eigenvectors but this would require a very large number of fields, as seen from the overlap matrix $(Y_i, \mathcal{Y}_j)$.

4.2 Dilaton and p-form KK modes

The dilaton is a particular case of the previous discussion. After expanding around τ_0, the mass matrix reads

$$M^{(\tau)2}_{I_1 J_1} = \frac{1}{4\mathrm{Im}\,\tau_0\, V_W}\int d^6y\sqrt{\tilde{g}_6}\, Y_{I_1}(y)\,\tilde{\nabla}^2 Y_{J_1}(y) = \mathcal{M}^{kk}_\tau\,\lambda^2_{I_1}\,. \tag{4.6}$$

Therefore,

$$\mathcal{L}^{(\tau)} = \mathcal{M}^{kk}_\tau \sum_{I_1} t^{I_1}(\Box + \lambda^2_{I_1})\bar{t}^{I_1}\,. \tag{4.7}$$

Similarly, we can write down the mass term for the vector coming from C_4,

$$M^{(V)2}_{IJ}\, V^I \wedge \star_4 V^J \tag{4.8}$$

where

$$M^{(V)2}_{IJ} = \frac{1}{4\,V_W}\int_M e^{4A}\, d\chi_I(y) \wedge \star_6 d\,\chi_J(y)\,. \tag{4.9}$$

Notice that, while the field space metric for the vector V_μ is unwarped, the warp factor enters into the mass matrix. Therefore this sector also exhibits light bound states, much as in the scalar field discussion.

4.3 Mass terms from dimensional reduction

The KK masses for the metric fluctuations follow from the effective action

$$S_{eff} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\,\sqrt{g}\left[(\delta g)^{\mu\nu}\delta G_{\mu\nu} + (\delta g)^{mn}\delta G_{mn}\right] \tag{4.10}$$

if we consider the metric fluctuations (3.7), but with variations being space-time independent. The conformal rescaling used to explain why there are

no spacetime derivative mixings between the graviton and internal metric modes does not rule out mass mixings of the form $\delta_K g\, \delta\tilde{g}$. Therefore we need to consider both types of fluctuations simultaneously.

The full computation is relegated to Appendix B. After making use of the constraints in (2.21) and (2.22), the mass terms simplify to

$$S_{mass} = \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{\tilde{g}_6} \Big[\frac{1}{2} \delta_K g_{\mu\nu} \tilde{\nabla}^2 (\delta_K g^{\mu\nu} - \eta^{\mu\nu} \delta_K g) - \delta\tilde{g}^{mn} \delta\tilde{G}_{mn} + \frac{1}{2} \delta_K g \delta\tilde{R} \Big] \,. \tag{4.11}$$

The first two terms give rise to geometric KK masses for the graviton and internal metric, while the last one mixes these massive sectors. We conclude that there are no mixing with the metric moduli, which satisfy $\delta\tilde{G}_{mn} = 0$. It is easily seen that $\delta\tilde{G}_{mn} = 0$ implies $\delta\tilde{R} = 0$ for a background unwarped metric which is Ricci-flat, i.e., $\tilde{R} = 0$.

Eq. (4.11) shows massive gravitons coupled to KK modes from the internal metric. This has a natural interpretation as a Higgs-type mechanism triggered by the spontaneous breaking of ten dimensional diffeomorphism invariance ($\langle g_{\mu\nu} \rangle$ and $\langle g_{mn} \rangle$ are nonzero). For instance, in the original Kaluza-Klein compactification on $\mathbb{R}^{(3,1)} \times S^1$, the infinite tower of massive spin 2 fields comes from combining the 4d gravitons plus Goldstone modes of spin 0 (from g_{55}) and spin 1 (from $g_{\mu 5}$) [59]. As in the gauge theory case, it should be possible to represent the massive states by a gauge invariant field combining the states of helicity 0, 1 and 2. This was done for the S^1 case in [60]. It would be very interesting to extend that analysis to the warped compactifications discussed here, and also to provide an explicitly supersymmetric construction [61].

5 Flux-induced Masses and Mixing

So far we have analyzed, in turns, how the different degrees of freedom propagate in spacetime, and then what is the mass structure that they inherit from the underlying warped geometry. The last missing piece in the analysis is given by the effect of background fluxes.

In truncations to the zero mode sector, the main role of these quantized fluxes is to lift the complex moduli, via the Gukov-Vafa-Witten superpotential. It is well understood how this contribution arises in unwarped scenarios but, as expected, the presence of warping introduces many new subtleties. For instance, at constant warping the Chern-Simons term is topological and does not contribute [18]. However, a warp factor introduces a nontrivial moduli dependence, from the relation $C_4 = e^{4A} d^4x$.

In [29], it was argued that the GVW superpotential in the presence of warping is not modified. However, this analysis did not take into account the warp factor variation and the CS contribution. On the other hand, the analysis of [26] was consistent with a GVW-type superpotential even in the presence of warping. Their approach is based on a direct ten dimensional analysis, where the potential is identified as the time component of the Einstein tensor fluctuation. We would like to understand what is the 4d role of this, and so we present a derivation based on the compactified effective action (2.24). In this section, we will also analyze the effect of fluxes on KK modes.

5.1 No mixing with p-form modes

Before starting our analysis of the flux potential, we show here that there are no mass mixings between the complex moduli S and KK modes coming from (B_2, C_2, C_4). After dimensional reduction, these 10d forms give 4d forms of various ranks. The first point to note is that, due to Lorentz invariance, the scalar field S can only mix with the zero forms; hence we restrict our attention to them.

First consider possible mixings coming from C_4 and the self-dual term. To account for self-duality, we set $\tilde{F}_5 = dC_4$ and multiply by two the terms where C_4 appears. After eliminating half of the degrees of freedom, the remaining KK modes from C_4 which contribute to $\tilde{F}_5^2$ term are either 1 and 2-forms in space-time, which cannot lead to mixing with moduli by Lorentz invariance, or 0-forms. Explicit computation shows that the scalar coming from C_4 does not lead to mixing.

Next, the bilinear terms involving S and the zero forms from (B_2, C_2) come from combining the $|G_3|^2$ and CS terms, yielding the usual term

$$S_{mix} = -\frac{1}{4\kappa_{10}^2 \operatorname{Im}\tau} \int G_3 \wedge \left(\star_{10} \bar{G}_3 + iC_4 \wedge \bar{G}_3 \right). \tag{5.1}$$

Here we assume a constant dilaton background; mixings with the dilaton KK modes will be analyzed momentarily. Expanding in a complete basis of internal two forms $\omega_A(y)$, the KK mode contribution to the 3-form reads

$$\delta G_3^{KK} = d\Big([c_A(x) - \tau b_A(x)]\, \omega_A(y) \Big)\,,$$

where a sum over A is implicit. If $\omega_A \in H^2(M)$, we recover the usual four dimensional zero modes which do not mix with S. Here we are interested in the massive modes, for which ω_A is not closed. Replacing in (5.1) and

expanding to quadratic order in the fields, we have (note that there are no quadratic terms with spacetime derivatives),

$$S_{mix} = -\frac{1}{4\kappa_{10}^2\,\mathrm{Im}\,\tau}\int d^4x\,(c_A - \tau b_A)\bar{S}\int_M d\omega_A(x)\wedge\partial_{\bar{S}}\Big(e^{4A}[\star_6\bar{G}_3 + i\bar{G}_3]\Big)\,. \tag{5.2}$$

Under a complex moduli fluctuation, the G_3 equation of motion implies that $\bar{\Lambda} = e^{4A}[\star_6\bar{G}_3 + i\bar{G}_3]$ is closed, so $S_{mix} = 0$ after integrating by parts.[4]

5.2 Flux-induced mass terms

The flux-induced masses for metric moduli and KK modes follow from Eq. (2.24). This involves computing the fluctuated energy momentum tensors from G_3 and F_5, and then contracting with the fluctuated metric. Also, recalling that we are working in backgrounds satisfying $e^{4A} = \alpha$, one gets extra pieces coming from the gravitational part δG_{MN}, which depends on the warp factor. Furthermore, following section 2.2, the equation of motion for α has to be imposed as a constraint, and this introduces flux dependence.

It turns out that the computation may be done including the moduli (the relevant ones here are the complex moduli and axio-dilaton) and their KK modes, in a symmetric way; refer to the Appendix for more details. In summary, the flux induced mass terms including moduli and KK modes are

$$\begin{aligned} S_{flux} &= -\frac{1}{2\kappa_{10}^2}\int d^4x\int d^6y\,\sqrt{\tilde{g}_6}\;e^{-2A}\left\{|\delta_K\tau|^2\partial_\tau\partial_{\bar{\tau}}\Big(\frac{G_3\cdot\bar{G}_3}{24\mathrm{Im}\tau}\Big) + \right.\\ &\left. +\;\delta\tilde{g}_m^{\tilde{n}}\delta\left[\frac{1}{8\mathrm{Im}\tau}\left(G_{npq}\,\bar{G}^{mpq} - \frac{1}{6}\delta_n^m\,|G_3|^2\right)\right]\right\} \end{aligned} \tag{5.3}$$

where the variation 'δ' in the last line includes both the axio-dilaton and internal metric fluctuations. To make the result more compact, indices with tildes are raised with $\tilde{g}_{mn}$, while the ones without tildes are raised with $g_{mn} = e^{-2A}\tilde{g}_{mn}$.

Restricting to the zero mode sector, this result shows the usual lifting of the moduli by fluxes. However, we would like to stress that we are including KK modes as well, as can be seen by inserting the mode expansion (2.6) into (5.3). The general analysis of the Appendix shows that there are no flux masses for the graviton KK modes. Further, one may check that only the traceless parts of the metric fluctuations are lifted by the fluxes. This is the familiar statement that Kähler moduli are not stabilized at this level, but it also implies that the trace part of the massive modes does not couple to the fluxes.

[4] In particular, $\bar{\Lambda}$ is a linear combination of Ω and $\bar{\chi}_S$.

5.3 Computation of the potential to all orders

One very interesting consequence of Eq. (5.3) is that the flux contribution may mix the zero modes with the massive fluctuations. It is very important to understand such mixings, since so far all the other terms in our effective action do not exhibit this effect (at least to quadratic order).

Unfortunately, S_{flux} presents a rather complicated structure and it seems that statements about mixings will depend strongly on the particular background, with the corresponding flux choice and form of $\delta\tilde{g}_{mn}$. Nevertheless, we now describe an alternative approach for finding S_{flux} which may be better suited for answering these sorts of questions.

The method is based on two observations: first, to compute the potential it is enough to consider space-time independent fluctuations. Also, the expression as a power series (2.24) is only necessary to identify the 'geometrical' KK masses. In order to find the flux potential such terms may be set to zero, and an appropriate use of the 10d equations of motion gives us an answer to all orders in the fluctuations.

This is in fact the spirit of the original GKP derivation [18] or the more detailed approach of [29]. However, for a nontrivial warp factor some terms would be missing in their derivation, and we also want to include KK modes.

The terms contributing to the potential are

$$S_{flux} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\left\{R - \frac{G_3\cdot\bar{G}_3}{12\mathrm{Im}\tau} - \frac{\tilde{F}_5^2}{480}\right\} - \frac{i}{8\kappa_{10}^2}\int\frac{C_4\wedge G_3\wedge\bar{G}_3}{\mathrm{Im}\tau}\,. \tag{5.4}$$

First, the Ricci scalar part has the form

$$\int d^{10}x\sqrt{-g}R = \int d^4x\int d^6y\,\sqrt{g_6}\,\left[-8e^{4A}(\nabla A)^2+\ldots\right]\,, \tag{5.5}$$

where the dots refer to terms induced by the KK modes, which are related to their geometric masses and do not depend on moduli. The flux dependence here comes from the equation of motion

Next, the G_3 term is already in the desired form. Finally, after integrating by parts and using the Bianchi identity for the 5-form, the $\tilde{F}_5^2$ and CS terms give

$$\int d^{10}x\,\sqrt{-g}\,\frac{\tilde{F}_5^2}{480} + \frac{i}{4}\int\frac{C_4\wedge G_3\wedge\bar{G}_3}{\mathrm{Im}\tau} \to \frac{i}{4}\int d^4x\int\frac{e^{4A(y)}}{\mathrm{Im}\,\tau}\,G_3\wedge\bar{G}_3\,.$$

Combining these contributions, we arrive to

$$S_{flux} = -\frac{1}{4\kappa_{10}^2}\int d^4x\int\frac{e^{4A}}{\mathrm{Im}\,\tau}\,G_3\wedge\left(\star_6\,\bar{G}_3 + i\bar{G}_3\right). \tag{5.6}$$

As a check, the second order variation of this expression reproduces our previous result Eq. (5.3).

Summarizing, Eq. (5.6) gives the full flux potential for the metric fluctuations including KK modes. This has the same form as the potential including only complex moduli. There are, however, two differences. This expression is valid including axio-dilaton fluctuations, while the original derivation set τ to a constant. Further, the massive metric modes are encoded in the Hodge star. One cannot use the method of [29] to obtain the GVW superpotential from here, since for arbitrary massive fluctuations we do not know which are the 3-forms with definite self-duality properties under $\star_6$. We plan to analyze these issues in a future work.

Acknowledgements

We thank F. Denef, R. Essig, A. Frey, S. Giddings, J. Gray, T. Grimm, K. Intriligator, S. Kachru, S. Klevtsov, H. Lu, A. Nacif, P. Ouyang, M. Porrati, K. van den Broek, T. van Riet and K. Sinha for helpful discussions and comments. G.S. and B.U. are supported in part by NSF CAREER Award No. PHY-0348093, DOE grant DE-FG-02-95ER40896, a Research Innovation Award and a Cottrell Scholar Award from Research Corporation. M.R.D. is supported in part by DOE grant DE-FG02-96ER40959. G.T. is supported as a Research Assistant in the Rutgers Department of Physics.

A Axial gauge equations for time dependent moduli

As discussed in Section 2.1, when the internal metric moduli are space-time dependent there can be additional fluctuations of the 10-dimensional metric which are proportional to spacetime derivatives of the moduli [26]

$$\delta_c ds^2 = 2\partial_\mu\partial_\nu u^I(x)e^{2A}K_I(y)dx^\mu dx^\nu + 2e^{2A}B_{Im}(y)\partial_\mu u^I(x)dx^\mu dy^m \ . \quad \text{(A.1)}$$

In general, these fluctuations can be gauged away by an appropriate gauge choice

$$\begin{aligned}\epsilon_\mu &= -\partial_\mu u^I(x)e^{2A}K_I(y) && \text{(A.2)}\\ \epsilon_m &= u^I(x)\partial_m\big(e^{2A}K_I(y)\big) - 2e^{2A}B_{Im}(y)u^I(x)\,. && \text{(A.3)}\end{aligned}$$

The only remaining gauge transformations allowed that preserve this gauge choice are spacetime independent internal diffeomorphisms $\partial_\mu\epsilon_m = 0$ and

pure four-dimensional diffeomorphisms $\partial_m \epsilon_\mu = 0$. Note that this gauge transformation induces a non-zero time dependent transformation of the fluctuated internal metric, $\delta\tilde{g}_{mn} \to \delta\tilde{g}_{mn} + 2\nabla_{(m}\epsilon_{n)}$, so that if the space-time dependent fluctuation started in transverse traceless gauge, it will no longer remain so.

One of the special features about GKP backgrounds that simplified our calculations is that the flux sector does not get modified when the moduli are promoted to space-time dependent fields. Therefore it's worth describing this in detail.

Promoting the moduli to spacetime dependent fields leads to the possibility of fluctuations in the p-form fluxes of the form [26]

$$\delta_c(C_2 - \tau B_2) = du^I \wedge T_I \;,\; \delta_c C_4 = du^I \wedge S_I^{(3)} + \star_4 du^I \wedge S_I \,. \qquad \text{(A.4)}$$

These fluctuations are found as solutions to the fluctuated equations of motion

$$d\big[\delta_I(\star_{10} G_3 - iC_4 \wedge G_3)\big] = 0\,, \qquad \text{(A.5)}$$

$$d\delta_I \tilde{F}_5 = \delta_I\left(H_3 \wedge F_3\right) + 2\kappa_{10}^2 T_{D3}\delta_I(\rho_3^{loc})\,, \qquad \text{(A.6)}$$

where $\delta_I := u^I(x)\partial_I$. The fluctuations are subject to the self-duality relation $\delta\tilde{F}_5 = \delta(\star_{10}\tilde{F}_5)$.

We will take the localized sources to be far away from the cycles on which the moduli are localized, so that $\delta(\rho_3^{loc}) \approx 0$. For instance, for a D3 brane at a distance $y = y_0$ from a warped throat with scale $e^{A_{min}} \sim \Lambda$, these corrections are suppressed by $\Lambda^4/|y_0|^4$. More concretely, in the embedding of the KS solution in a compact 3-fold given in [18], $|y_0|^6 \sim V_{(6)}$. However, it would also be interesting to understand the effects of localized sources inside the throat, for applications to sequestering and supersymmetry breaking (see, for instance, [8, 43, 63]).

Solving the flux equations of motion (A.5, A.6), subject to the self-duality constraint, it is easy to see that the internal space part of the fluctuations (A.4) are constant, $dT_I = dS_I = 0$ (we also find that $S_I^{(3)} = 0$). Since these are closed one forms on a Calabi-Yau space, they must be exact, which implies that they can be gauged away. This is the gauge choice we have used in this work.

There are situations where this is not the case, and having time-dependent moduli has physical effects on the p-forms; a related example is analyzed in [39]. Here, the reason why we can set $T_I = S_I = S_I^{(3)} = 0$ is that the equations of motion (A.5) and (A.6) are actually *time-independent*. Therefore, promoting the moduli to fields does not induce changes in this sector.

B Warped KK mode effective action

Fluctuated equations of motion

We begin by collecting the relevant supergravity formulas for the ten dimensional fluctuations, following [26]. The fluctuated Einstein's equations are

$$\delta G^M_N = \kappa_{10}^2 \delta T^M_N \,, \tag{B.1}$$

where raising one of the indices simplifies the computation of (2.20) by eliminating derivatives of the warp factor. First,

$$\begin{aligned}
\delta(G^\mu_\nu) \;&=\; \delta^\mu_\nu \delta\big[-\frac{1}{2}e^{2A}\tilde{R} + \frac{3}{4}e^{-6A}\partial_m e^{4A}\partial^{\tilde{m}} e^{4A} - \frac{1}{2}e^{-2A}\tilde{\nabla}^2 e^{4A}\big] + \\
&+\; e^{-2A}\delta_K G^{(4)\hat{\mu}}_{\ \ \nu} - \frac{1}{2}e^{2A}\tilde{\nabla}^2(\delta_K g^{\hat{\mu}}_\nu - \delta^\mu_\nu \delta_K g)
\end{aligned} \tag{B.2}$$

where $\delta_K g := \eta^{\mu\nu}\delta_K g_{\mu\nu}$ and $\delta_K G^{(4)}$ is the four-dimensional Einstein's tensor for $\eta_{\mu\nu} + \delta_K g_{\mu\nu}$. On the other hand,

$$\begin{aligned}
\delta(G^m_n) \;&=\; \delta\big[e^{2A}(\tilde{G}^{\tilde{m}}_n + \frac{1}{4}e^{-8A}\delta^{\tilde{m}}_n \partial_p e^{4A}\partial^{\tilde{p}} e^{4A} - \frac{1}{2}e^{-8A}\partial_n e^{4A}\partial^{\tilde{m}} e^{4A})\big] \\
&+\; \frac{1}{2}\big[-e^{-2A}\tilde{\nabla}^{\tilde{(}m}(e^{4A}\partial_{n)}\delta_K g) + \delta^m_n \tilde{\nabla}^{\tilde{p}}(e^{2A}\partial_p \delta_K g)\big] + \\
&-\; \frac{1}{2}\delta^m_n e^{-2A}\delta_K R^{(4)} - \frac{1}{2}e^{-2A}\Box\delta\tilde{g}^{\tilde{m}}_n + \frac{1}{4}\delta^m_n e^{-2A}\Box\delta\tilde{g}\,.
\end{aligned} \tag{B.3}$$

The index notation is $\delta_K g^{\hat{\mu}}_\nu := \eta^{\mu\lambda}\,\delta_K g_{\lambda\nu}$ and $\delta\tilde{g}^{\tilde{m}}_n := \tilde{g}^{mp}\,\delta\tilde{g}_{pn}$, and similarly for other tensors. Finally,

$$\delta G^\mu_m = e^{-2A}\partial^{\hat{\mu}}\big(-\frac{1}{4}\partial_m\delta\tilde{g} - 2\partial^{\tilde{p}}A\,\delta\tilde{g}_{mp} + \frac{1}{2}\,\tilde{\nabla}^{\tilde{p}}\,\delta\tilde{g}_{mp}\big)\,. \tag{B.4}$$

The energy momentum tensor has contributions from the three and five forms, and from other local sources (3-branes and orientifolds): $T_{MN} = T^{(5)}_{MN} + T^{(3)}_{MN} + T^{loc}_{MN}$. A straightforward computation gives

$$T^{(5)\mu}_\nu + T^{(3)\mu}_\nu \;=\; -\frac{1}{4\kappa_{10}^2}\,\delta^\mu_\nu\big(e^{-6A}\,\partial_m\alpha\partial^{\tilde{m}}\alpha + \frac{G_3\cdot\bar{G}_3}{8\mathrm{Im}\,\tau}\big)\,, \tag{B.5}$$

$$\begin{aligned}
T^{(5)n}_m + T^{(3)n}_m \;&=\; \frac{1}{2\kappa_{10}^2}\,\big[-e^{-6A}(\partial_m\alpha\partial^{\tilde{n}}\alpha - \frac{1}{2}\delta^n_m\partial_p\alpha\partial^{\tilde{p}}\alpha) + \\
&+\; \frac{1}{4\mathrm{Im}\,\tau}(G_{mpq}\bar{G}^{npq} + G^n_{\ pq}\bar{G}_m^{\ pq} - \frac{1}{3}\delta^n_m G_3\cdot\bar{G}_3)\big]\,.
\end{aligned} \tag{B.6}$$

Besides the fluctuated Einstein's equations, the three and five form equations of motion become *constraints* on the four dimensional fluctuations:

$$\delta\big(\tilde{\nabla}^2\alpha - 2e^{-6A}\partial_m\alpha\partial^{\tilde{m}}e^{4A}\big) = \delta\big(ie^{2A}\frac{G_{mnp}\,(\star_6\bar{G})^{mnp}}{12\,\mathrm{Im}\,\tau} + 2\kappa_{10}^2 e^{2A}\,T_3\rho_3^{loc}\big) \quad (B.7)$$

$$d[e^{4A}\,(\delta\,\star_6\,G_3)] = 0 \quad (B.8)$$

where in the last line the fact that the background is ISD was used. Another constraint follows from fluctuating the self-duality condition $\star_{10}\tilde{F}_5 = \tilde{F}_5$, which is equivalent to $\star_{10}\,dC_4 = B_2\wedge F_3$. In our case, under a metric fluctuation, fluxes stay fixed at their quantized values, so we get

$$\delta[\star_{10}\,dC_4] = 0\,. \quad (B.9)$$

In this work we will restrict to fluctuations that preserve the form of the BPS condition $\alpha = e^{4A}$, i.e., $\delta(\alpha - e^{4A}) = 0$.

These properties of the BPS flux compactifications that we are considering are at the root of many restrictions in the possible mixings. The $(\mu\nu)$ fluctuated Einstein equation simplifies to

$$\begin{aligned}\delta G^\mu_\nu - \kappa_{10}^2\delta T^\mu_\nu &= e^{-2A}\,\delta_K G^{(4)\,\hat{\mu}}_\nu - \frac{1}{2}e^{2A}\,\tilde{\nabla}^2\big(\delta_K g^{\hat{\mu}}_\nu - \delta^\mu_\nu\,\delta_K g\big) + \\ &- \frac{1}{2}\delta^\mu_\nu e^{2A}\,\delta\tilde{R} - \delta^\mu_\nu\kappa_{10}^2 T_3\delta\rho_3^{loc} - \kappa_{10}^2\,\delta T^\mu_{loc\,\nu}\,. \quad (B.10)\end{aligned}$$

We remind the reader that hats denote indices raised with $\eta_{\mu\nu}$. Similarly, in the (mn) component all the α and A variations cancel, yielding

$$\begin{aligned}\delta G^m_n - \kappa_{10}^2\delta T^m_n &= e^{2A}\,\delta\,\tilde{G}^{\tilde{m}}_n - \delta\big[\frac{1}{8\mathrm{Im}\,\tau}\big(2G_{mpq}\bar{G}^{npq} - \frac{1}{3}\delta^n_m G_3\cdot\bar{G}_3\big)\big] + \\ &- \frac{1}{2}e^{-2A}\Box\,\delta\tilde{g}^{\tilde{m}}_n + \frac{1}{4}e^{-2A}\delta^m_n\,\Box\,\delta\tilde{g} - \frac{1}{2}e^{-2A}\,\tilde{\nabla}^{\tilde{m}}(e^{4A}\partial_n\delta_K g) + \\ &+ \frac{1}{2}\delta^m_n\tilde{\nabla}^{\tilde{p}}(e^{2A}\partial_p\delta_K g) - \frac{1}{2}e^{-2A}\delta^m_n\delta_K R^{(4)\,\hat{\mu}}_\mu - \kappa_{10}^2\delta T^m_{loc\,n} \quad (B.11)\end{aligned}$$

where we already combined the symmetric terms, since this is going to be contracted with δg_{mn}; tildes refer to indices raised with $\tilde{g}_{mn}$. Finally, the (μm) equation gives the constraint

$$-\frac{1}{4}\partial_m\delta\tilde{g} - 2\partial^{\tilde{p}}A\,\delta\tilde{g}_{mp} + \frac{1}{2}\,\tilde{\nabla}^{\tilde{p}}\,\delta\tilde{g}_{mp} = 0\,. \quad (B.12)$$

Derivation of the Effective Action

Using the previous fluctuated equations of motion gives, after contracting (3.7) with (B.2) and (B.3),

$$\begin{aligned} S_{eff} &= -\frac{1}{2}\int d^4x \int d^6y\,\sqrt{\tilde{g}_6}\Big[e^{-4A}\,\delta_K g^{\hat{\nu}}_{\mu}\,\delta_K G^{(4)\,\hat{\mu}}_{\nu} - \frac{1}{2}\delta_K g^{\hat{\nu}}_{\mu}\tilde{\nabla}^2\big(\delta_K g^{\hat{\mu}}_{\nu} - \delta^{\mu}_{\nu}\,\delta_K g\big) + \\ &- \frac{1}{2}e^{-4A}\,\delta\tilde{g}^{\tilde{n}}_{m}\Box\,\delta\tilde{g}^{\tilde{m}}_{n} + \delta\tilde{g}^{\tilde{n}}_{m}\,\delta\tilde{G}^{\tilde{m}}_{n} - \frac{e^{-2A}}{8\,\mathrm{Im}\,\tau}\delta\tilde{g}^{\tilde{n}}_{m}\big(2G_{npq}\,\delta\bar{G}^{mpq} - \frac{1}{3}\delta^{m}_{n}\,\delta[G_3\cdot\bar{G}_3]\big) \\ &+ \frac{1}{2}(-\delta_K g\,\delta\tilde{R} + \frac{3}{4}\delta_K g\,\tilde{\nabla}^2\delta\tilde{g}) + \frac{1}{2}e^{-2A}\big\{-e^{-2A}\delta\tilde{g}^{\tilde{n}}_{m}\tilde{\nabla}^{\tilde{m}}(e^{4A}\partial_n\delta_K g) + \\ &- \frac{1}{2}\delta\tilde{g}\tilde{\nabla}^{\tilde{p}}(e^{2A}\partial_p\delta_K g) + \frac{1}{4}e^{-2A}\delta\tilde{g}\tilde{\nabla}^{\tilde{m}}(e^{4A}\partial_m\delta_K g)\big\} - \kappa^2_{10}\,e^{-2A}\delta g\,\delta T_{loc}\Big] \end{aligned} \quad \text{(B.13)}$$

where the last term stands for

$$\delta g\,\delta T_{loc} := T_3\,\delta_K g\,\delta\rho^{loc}_3 + \delta_K g^{\hat{\nu}}_{\mu}\,\delta T^{\mu}_{loc\,\nu} - \frac{1}{4}\delta\tilde{g}\,\delta T^{m}_{loc\,m} + \delta\tilde{g}^{\tilde{n}}_{m}\,\delta T^{m}_{loc\,n}\,. \quad \text{(B.14)}$$

To avoid cluttering, the dilaton fluctuations have not been included; they will be considered shortly.

The final step in the computation is to impose the constraint (B.12), after which we obtain

$$S_{eff} = S^{(KK)}_{eff} + S^{(S)}_{eff} + S^{(\tau)}_{eff} + S_{mix} \quad \text{(B.15)}$$

where

$$S^{(KK)}_{eff} = -\frac{1}{2}\int d^4x\int d^6y\,\sqrt{\tilde{g}_6}\big[e^{-4A}\,\delta_K g^{\nu}_{\mu}\,\delta_K G^{(4)\,\mu}_{\nu} - \frac{1}{2}\delta_K g^{\nu}_{\mu}\tilde{\nabla}^2\big(\delta_K g^{\mu}_{\nu} - \delta^{\mu}_{\nu}\,\delta_K g\big)\big] \quad \text{(B.16)}$$

is the usual graviton KK mode action, while the one for internal fluctuations is

$$S^{(S)}_{eff} = -\frac{1}{2}\int d^4x\int d^6y\,\sqrt{\tilde{g}_6}\,\big[-\frac{1}{2}e^{-4A}\,\delta\tilde{g}^{mn}\Box\,\delta\tilde{g}_{mn} + \delta\tilde{g}^{mn}\,\delta\tilde{G}_{mn}\big] \quad \text{(B.17)}$$

and the dilaton part reads

$$S^{(\tau)}_{eff} = \int d^4x\int d^6y\sqrt{\tilde{g}_6}\big[\frac{1}{4(\mathrm{Im}\tau_0)^2}e^{-4A}\delta_K\bar{\tau}\Box\delta_K\tau + \frac{1}{4(\mathrm{Im}\tau_0)^2}\delta_K\bar{\tau}\tilde{\nabla}^2\delta_K\tau\big]\,. \quad \text{(B.18)}$$

The mixing term has the structure

$$S_{mix} = \frac{1}{2}\int d^4x\int d^6y\,\sqrt{\tilde{g}_6}\,\Big[\frac{1}{2}\delta_K g\,\delta\tilde{R} + \kappa^2_{10}\,e^{-2A}\delta g\,\delta T_{loc}\Big] + S_{flux}\,, \quad \text{(B.19)}$$

where the flux contribution is given by

$$
\begin{aligned}
S_{flux} &= -\int d^4x \int d^6y\,\sqrt{\tilde{g}_6}\; e^{-2A}\left\{|\delta_K\tau|^2\partial_\tau\partial_{\bar\tau}(\frac{G_3\cdot\bar{G}_3}{24\mathrm{Im}\tau})+\right.\\
&+ \left.\delta\tilde{g}^{\tilde{n}}_m\delta\left[\frac{1}{8\mathrm{Im}\tau}\left(G_{npq}\,\bar{G}^{mpq}-\frac{1}{6}\delta^m_n\,|G_3|^2\right)\right]\right\} \qquad \text{(B.20)}
\end{aligned}
$$

Where there is no confusion we have eliminated the hats and tildes, and in the last line the variation δ includes complex moduli and dilaton fluctuations. The flux contribution may in principle mix the complex structure zero modes with their KK excitations and the axio-dilaton KK modes.

C Summary of warped effective theory

Inserting the mode expansions of the fields into (B.15) and integrating over the internal space gives rise to the complete four dimensional effective action. Throughout the paper we computed the effective action in separate sections. Here we collect our expressions for the four dimensional effective Lagrangian for the complex and Kähler moduli, graviton KK modes, and dilaton KK modes:

$$
\begin{aligned}
\mathcal{L}_{eff} &= \mathcal{L}_u+\mathcal{L}_{KK}+\mathcal{L}_\tau+\mathcal{L}_{mix}\\
&= G^{(u)}\sum_{I_2}\bar{u}^{I_2}\left(\Box+\lambda^2_{I_2}\right)u^{I_2}+\mathcal{M}^{kk}\sum_{I_1}h^{\mu\nu}_{I_1}\left(E^{I_1}_{\mu\nu}+\lambda^2_{I_1}h^{I_1}_{\mu\nu}\right)\\
&\quad+\mathcal{M}^{kk}_\tau\sum_{I_1}\bar{t}^{I_1}\left(\Box+\lambda^2_{I_1}\right)t^{I_1}+\sum_{I_1,J_1}A_{I_1J_1}\bar{t}^{I_1}t^{J_1}-\sum_{I_1J_2}\gamma_{I_1J_2}\bar{u}^{J_2}h^{I_1}\\
&\quad+\sum_{I_2,J_2}(\alpha_{I_2J_2}-\beta_{I_2J_2})\,\bar{u}^{I_2}u^{J_2}+\sum_{I_2,J_1}B_{I_2J_1}\,u^{I_2}\bar{t}^{J_1}\,. \qquad \text{(C.1)}
\end{aligned}
$$

The various matrices here are defined as

$$
G^{(u)} = \frac{1}{4V_W}\int d^6y\sqrt{\tilde{g}_6}e^{-4A}\,Y^{I_2}_{mn}Y^{I_2\widetilde{mn}} \qquad \text{(C.2)}
$$

$$
\mathcal{M}^{kk} = -4(\mathrm{Im}\tau_0)^2\,\mathcal{M}^{kk}_\tau=\frac{1}{2V_W}\int d^6y\sqrt{\tilde{g}_6}e^{-4A}\,Y^{I_1}Y^{I_1} \qquad \text{(C.3)}
$$

$$
\alpha_{I_2J_2} = \frac{3}{8\mathrm{Im}\tau V_W}\int d^6y\sqrt{\tilde{g}_6}\,Y^{I_2}_{mn}(Y^{J_2\widetilde{mm'}}-\frac{1}{4}Y^{J_2\tilde{p}}_p\tilde{g}^{mm'})G^n_{pq}\bar{G}^{pq}_{m'} \qquad \text{(C.4)}
$$

$$\beta_{I_2 J_2} = \frac{1}{16\mathrm{Im}\tau V_W}\int d^6y\sqrt{\tilde{g}_6}\,Y^{I_2}(Y^{J_2\widetilde{rr'}} - \frac{1}{4}Y_p^{J_2\tilde{p}}\tilde{g}^{rr'})G_{pqr}\bar{G}_{r'}^{pq} \tag{C.5}$$

$$\gamma_{I_1 J_2} = \frac{1}{4V_W}\int d^6y\sqrt{\tilde{g}_6}\,Y^{I_1}\tilde{\nabla}^m\tilde{\nabla}^n Y^{I_2}_{mn} \tag{C.6}$$

$$A_{I_1 J_1} = -\frac{1}{48V_W}\int d^6y\sqrt{\tilde{g}_6}\;e^{-2A}\frac{|G_3|^2}{(\mathrm{Im}\tau_0)^3}\,Y^{I_1}Y^{J_1} \tag{C.7}$$

$$B_{I_2 J_1} = \frac{1}{4V_W}\int d^6y\sqrt{\tilde{g}_6}e^{-2A}Y^{I_2\widetilde{mn}}\bar{Y}^{J_1} \left[\frac{G_{mpq}\bar{G}_n^{pq} - \frac{1}{3}g_{mn}|G_3|^2}{(\mathrm{Im}\tau_0)^2} + \frac{1}{\mathrm{Im}\tau_0}(\tau H_{mpq}H_n^{pq} - F_{mpq}H_n^{pq})\right] \tag{C.8}$$

In the limit of weak warping $e^A \approx 1$, this reduces to the usual unwarped 4-dimensional effective theory.

References

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[2] F. Cachazo, K. A. Intriligator and C. Vafa, Nucl. Phys. B **603**, 3 (2001) [arXiv:hep-th/0103067]. R. Dijkgraaf and C. Vafa, arXiv:hep-th/0208048.

[3] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[4] A. Ceresole, G. Dall'Agata, R. D'Auria and S. Ferrara, Phys. Rev. D **61**, 066001 (2000) [arXiv:hep-th/9905226]. A. Ceresole, G. Dall'Agata and R. D'Auria, JHEP **9911**, 009 (1999) [arXiv:hep-th/9907216].

[5] M. Berg, M. Haack and W. Mueck, Nucl. Phys. B **736**, 82 (2006) [arXiv:hep-th/0507285]. M. Berg, M. Haack and W. Mueck, Nucl. Phys. B **789**, 1 (2008) [arXiv:hep-th/0612224].

[6] A. Dymarsky and D. Melnikov, arXiv:0710.4517 [hep-th]. M. K. Benna, A. Dymarsky, I. R. Klebanov and A. Solovyov, arXiv:0712.4404 [hep-th]. M. K. Benna, A. Dymarsky and I. R. Klebanov, JHEP **0708**, 034 (2007) [arXiv:hep-th/0612136].

[7] S. S. Gubser, C. P. Herzog and I. R. Klebanov, JHEP **0409**, 036 (2004) [arXiv:hep-th/0405282].

[8] S. Kachru, J. Pearson and H. L. Verlinde, JHEP **0206**, 021 (2002) [arXiv:hep-th/0112197]. O. DeWolfe, S. Kachru and M. Mulligan, arXiv:0801.1520 [hep-th].

[9] M. Aganagic, C. Beem, J. Seo and C. Vafa, Nucl. Phys. B **789**, 382 (2008) [arXiv:hep-th/0610249]. J. J. Heckman, J. Seo and C. Vafa, JHEP **0707**, 073 (2007) [arXiv:hep-th/0702077].

[10] M. R. Douglas, J. Shelton and G. Torroba, arXiv:0704.4001 [hep-th].

[11] S. Franco and A. M. Uranga, JHEP **0606**, 031 (2006) [arXiv:hep-th/0604136]. S. Franco, A. Hanany, F. Saad and A. M. Uranga, JHEP **0601**, 011 (2006) [arXiv:hep-th/0505040]. S. Franco, I. Garcia-Etxebarria and A. M. Uranga, JHEP **0701**, 085 (2007) [arXiv:hep-th/0607218].

[12] D. E. Diaconescu, A. Garcia-Raboso and K. Sinha, JHEP **0606**, 058 (2006) [arXiv:hep-th/0602138]. D. Malyshev, arXiv:0705.3281 [hep-th]. K. Sinha, arXiv:0709.2932 [hep-th].

[13] K. Intriligator, N. Seiberg and D. Shih, JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].

[14] R. Argurio, M. Bertolini, S. Franco and S. Kachru, JHEP **0701**, 083 (2007) [arXiv:hep-th/0610212]. R. Argurio, M. Bertolini, S. Franco and S. Kachru, JHEP **0706**, 017 (2007) [arXiv:hep-th/0703236].

[15] R. Essig, K. Sinha and G. Torroba, JHEP **0709**, 032 (2007) [arXiv:0707.0007 [hep-th]]. M. Buican, D. Malyshev and H. Verlinde, arXiv:0710.5519 [hep-th].

[16] A. Giveon and D. Kutasov, Nucl. Phys. B **778**, 129 (2007) [arXiv:hep-th/0703135]. A. Giveon and D. Kutasov, JHEP **0802**, 038 (2008) [arXiv:0710.1833 [hep-th]].

[17] M. Dine, G. Festuccia, A. Morisse and K. van den Broek, arXiv:0712.1397 [hep-th].

[18] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D **66**, 106006 (2002) [arXiv:hep-th/0105097].

[19] M. Grana, Phys. Rept. **423**, 91 (2006) [arXiv:hep-th/0509003].

[20] M. R. Douglas and S. Kachru, Rev. Mod. Phys. **79**, 733 (2007) [arXiv:hep-th/0610102]. F. Denef, M. R. Douglas and S. Kachru, Ann. Rev. Nucl. Part. Sci. **57**, 119 (2007) [arXiv:hep-th/0701050].

[21] F. Denef, arXiv:0803.1194 [hep-th].

[22] A. R. Frey, arXiv:hep-th/0308156.

[23] K. Becker and M. Becker, Nucl. Phys. B **477**, 155 (1996) [arXiv:hep-th/9605053].

[24] H. L. Verlinde, Nucl. Phys. B **580**, 264 (2000) [arXiv:hep-th/9906182].

[25] K. Dasgupta, G. Rajesh and S. Sethi, JHEP **9908**, 023 (1999) [arXiv:hep-th/9908088].

[26] S. B. Giddings and A. Maharana, Phys. Rev. D **73**, 126003 (2006) [arXiv:hep-th/0507158].

[27] I. Benmachiche and T. W. Grimm, Nucl. Phys. B **748**, 200 (2006) [arXiv:hep-th/0602241].

[28] P. Koerber and L. Martucci, JHEP **0708**, 059 (2007) [arXiv:0707.1038 [hep-th]].

[29] O. DeWolfe and S. B. Giddings, Phys. Rev. D **67**, 066008 (2003) [arXiv:hep-th/0208123].

[30] A. R. Frey and J. Polchinski, Phys. Rev. D **65**, 126009 (2002) [arXiv:hep-th/0201029].

[31] S. P. de Alwis, Phys. Rev. D **68**, 126001 (2003) [arXiv:hep-th/0307084]; A. Buchel, Phys. Rev. D **69**, 106004 (2004) [arXiv:hep-th/0312076];

[32] H. Kodama and K. Uzawa, JHEP 0603, 053 (2006) [arXiv:hep-th/0512104]; K. Koyama, K. Koyama and F. Arroja, Phys. Lett. B **641**, 81 (2006) [arXiv:hep-th/0607145].

[33] A. R. Frey and A. Maharana, JHEP **0608**, 021 (2006) [arXiv:hep-th/0603233].

[34] C. P. Burgess, P. G. Camara, S. P. de Alwis, S. B. Giddings, A. Maharana, F. Quevedo and K. Suruliz, arXiv:hep-th/0610255.

[35] G. Torroba, JHEP **0702**, 061 (2007) [arXiv:hep-th/0611002].

[36] S. Gukov, C. Vafa and E. Witten, Nucl. Phys. B **584**, 69 (2000) [Erratum-ibid. B **608**, 477 (2001)] [arXiv:hep-th/9906070].

[37] T. R. Taylor and C. Vafa, Phys. Lett. B **474**, 130 (2000) [arXiv:hep-th/9912152].

[38] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240].

[39] J. Gray and A. Lukas, Phys. Rev. D **70**, 086003 (2004) [arXiv:hep-th/0309096].

[40] P. Hoxha, R. R. Martinez-Acosta and C. N. Pope, Class. Quant. Grav. **17**, 4207 (2000) [arXiv:hep-th/0005172]; C. Pope, *Kaluza-Klein Theory*, http://faculty.physics.tamu.edu/pope/ ihplec.pdf.

[41] S. Dimopoulos, S. Kachru, N. Kaloper, A. E. Lawrence and E. Silverstein, Int. J. Mod. Phys. A **19**, 2657 (2004) [arXiv:hep-th/0106128].

[42] K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B **718**, 113 (2005) [arXiv:hep-th/0503216]; M. Gabella, T. Gherghetta and J. Giedt, Phys. Rev. D **76**, 055001 (2007) [arXiv:0704.3571 [hep-ph]].

[43] S. Kachru, L. McAllister and R. Sundrum, JHEP **0710**, 013 (2007) [arXiv:hep-th/0703105]; K. Choi, arXiv:0705.3330 [hep-th].

[44] S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, JCAP **0310**, 013 (2003) [arXiv:hep-th/0308055].

[45] D. Baumann, A. Dymarsky, I. R. Klebanov and L. McAllister, JCAP **0801**, 024 (2008) [arXiv:0706.0360 [hep-th]]; D. Baumann, A. Dymarsky, I. R. Klebanov, L. McAllister and P. J. Steinhardt, Phys. Rev. Lett. **99**, 141601 (2007) [arXiv:0705.3837 [hep-th]]; A. Krause and E. Pajer, arXiv:0705.4682 [hep-th]. For a recent review, see L. McAllister and E. Silverstein, Gen. Rel. Grav. **40**, 565 (2008) [arXiv:0710.2951 [hep-th]].

[46] L. Kofman and P. Yi, Phys. Rev. D **72**, 106001 (2005) [arXiv:hep-th/0507257].

[47] D. Chialva, G. Shiu and B. Underwood, JHEP **0601**, 014 (2006) [arXiv:hep-th/0508229].

[48] N. Barnaby, C. P. Burgess and J. M. Cline, JCAP **0504**, 007 (2005) [arXiv:hep-th/0412040]; A. R. Frey, A. Mazumdar and R. C. Myers, Phys. Rev. D **73**, 026003 (2006) [arXiv:hep-th/0508139]; X. Chen

and S. H. Tye, JCAP **0606**, 011 (2006) [arXiv:hep-th/0602136]; B. v. Harling, A. Hebecker and T. Noguchi, JHEP **0711**, 042 (2007) [arXiv:0705.3648 [hep-th]].

[49] H. Firouzjahi and S. H. Tye, JHEP **0601**, 136 (2006) [arXiv:hep-th/0512076].

[50] J. F. Dufaux, L. Kofman and M. Peloso, arXiv:0802.2958 [hep-th].

[51] G. Shiu, B. Underwood, K. M. Zurek and D. G. E. Walker, Phys. Rev. Lett. **100**, 031601 (2008) [arXiv:0705.4097 [hep-ph]].

[52] T. W. Grimm and J. Louis, Nucl. Phys. B **699**, 387 (2004) [arXiv:hep-th/0403067]. T. W. Grimm, Fortsch. Phys. **53**, 1179 (2005) [arXiv:hep-th/0507153].

[53] R. M. Wald, *General Relativity*, University of Chicago Press, 1984; chapter 7.

[54] C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D **63**, 065002 (2001) [arXiv:hep-th/0008151].

[55] P. Candelas and X. de la Ossa, Nucl. Phys. B **355**, 455 (1991).

[56] J. W. . York, Phys. Rev. Lett. **28** (1972) 1082. G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).

[57] D. E. Diaconescu, M. R. Douglas and J. Gomis, JHEP **9802**, 013 (1998) [arXiv:hep-th/9712230]. M. Aganagic and C. Beem, Nucl. Phys. B **796**, 44 (2008) [arXiv:0711.0385 [hep-th]].

[58] S. Weinberg, *The Quantum Theory of Fields*, volume III, Cambridge University Press, 2000.

[59] A. Salam and J. A. Strathdee, Annals Phys. **141**, 316 (1982). L. Dolan and M. J. Duff, Phys. Rev. Lett. **52**, 14 (1984).

[60] Y. M. Cho and S. W. Zoh, Phys. Rev. D **46**, 2290 (1992).

[61] W. D. I. Linch, M. A. Luty and J. Phillips, Phys. Rev. D **68**, 025008 (2003) [arXiv:hep-th/0209060]. T. Gregoire, M. D. Schwartz and Y. Shadmi, JHEP **0407**, 029 (2004) [arXiv:hep-th/0403224].

[62] M. R. Douglas and G. Torroba, to appear.

[63] O. Aharony, Y. E. Antebi and M. Berkooz, Phys. Rev. D **72**, 106009 (2005) [arXiv:hep-th/0508080].